Sections 3.5
The Reciprocal Family Function

-- Transform the reciprocal function
-- Use vertical & horizontal asymptotes to write the function
-- Write the equation of a transformed rational function
For a class party, the students will share the cost for the hall rental. Each student will also have to pay $8 for food. The cost of the hall rental is already graphed. What effect does the food cost have on the graph? Explain your reasoning.
Remember Transformations?

How do we:

- shift a graph up?
- shift a graph down?
- shift a graph to the left?
- shift a graph to the right?
- reflect a graph?
- stretch or shrink a graph vertically?
Functions that model inverse variation have the form \( f(x) = \frac{a}{x} \), where \( x \neq 0 \). They belong to a family whose parent is the \textbf{reciprocal function} \( f(x) = \frac{1}{x} \), where \( x \neq 0 \).

\textbf{Essential Understanding} Transformations of the parent reciprocal function include stretches, compressions (or shrinks), reflections, and horizontal and vertical translations.

\textbf{Key Concept} \quad \textbf{General Form of the Reciprocal Function Family}

The general form of a member of the reciprocal function family is \( y = \frac{a}{x - h} + k \), where \( x \neq h \).

The inverse variation functions, \( y = \frac{a}{x} \), are stretches, shrinks, and reflections of the parent reciprocal function, depending on the value of \( a \).

The graph of the parent reciprocal function \( y = \frac{1}{x} \) is shown at the right.

An \textbf{asymptote} is a line that a graph approaches as \( x \) or \( y \) increases in absolute value. The \( x \)- and \( y \)-axes are asymptotes of the parent reciprocal function.
What is the graph of \( y = \frac{8}{x}, \ x \neq 0 \)? Identify the \( x \)- and \( y \)-intercepts and the asymptotes of the graph. Also, state the domain and range of the function.
Problem 1

Got It?

a. What is the graph of \( y = \frac{12}{x} \)? Identify the \( x \)- and \( y \)-intercepts and the asymptotes of the graph. Also, state the domain and range of the function.

b. Reasoning Would the function \( y = \frac{6}{x} \) have the same domain and range as \( y = \frac{8}{x} \) or \( y = \frac{12}{x} \)? Explain.
Practice

Graph each function. Identify the $x$- and $y$-intercepts and the asymptotes of the graph. Also, state the domain and the range of the function.

1. $y = \frac{-3}{x}$

2. $y = \frac{10}{x}$
Each part of the graph of a reciprocal function is a **branch**. The branches of the parent function $y = \frac{1}{x}$ are in Quadrants I and III. Stretches and compressions of the parent function remain in the same quadrants. Reflections are in Quadrants II and IV.

\[
\begin{align*}
  y &= \frac{1}{x} \\
  y &= -\frac{1}{x}
\end{align*}
\]
Problem 2  Identifying a Reciprocal Function Transformation

For each given value of \( a \), how do the graphs of \( y = \frac{1}{x} \) and \( y = \frac{a}{x} \) compare? What is the effect of \( a \) on the graph?

A  \( a = 6 \)

B  \( a = 0.25 \)

C  \( a = -6 \)
Got It? For each given value of $a$, how do the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x}$ compare? What is the effect of $a$ on the graph?

a. $a = \frac{1}{2}$

b. $a = 2$

c. $a = -\frac{1}{2}$
You can translate any reciprocal function horizontally or vertically just as you can other functions.

**Key Concept  The Reciprocal Function Family**

**Parent function**

\[ y = \frac{1}{x}, \quad x \neq 0 \]

- Stretch (\(|a| > 1\))
- Shrink (\(0 < |a| < 1\))
  - Reflection (\(a < 0\)) across x-axis
- Translation (horizontal by \(h\); vertical by \(k\))
  - with vertical asymptote \(x = h\), and
  - horizontal asymptote \(y = k\)

**Combined**

\[ y = \frac{a}{x-h} + k; \quad x \neq h \]

When you graph a translated reciprocal function, a good first step is to draw the asymptotes.
What is the graph of $y = \frac{1}{x + 1} - 2$? Identify the domain and range.
Got It? What is the graph of \( y = \frac{1}{x - 4} + 6 \)? Identify the domain and range.
Practice Sketch the asymptotes and the graph of each function. Identify the domain and range.

5. \( y = \frac{1}{x} - 2 \)

6. \( y = \frac{-8}{x + 5} - 6 \)
Multiple Choice  This graph of a function is a translation of the graph of $y = \frac{2}{x}$. What is an equation for the function?

A  $y = \frac{2}{x+3} + 4$
B  $y = \frac{2}{x+3} - 4$
C  $y = \frac{2}{x-3} + 4$
D  $y = \frac{2}{x-3} - 4$
Got It? This graph of a function is a translation of the graph of $y = \frac{2}{x}$. What is an equation for the function?
Practice  Write an equation for the translation of $y = \frac{2}{x}$ that has the given asymptotes.

7. $x = 0$ and $y = 4$

8. $x = 4$ and $y = -8$
Additional Practice
Sketch the graph of each function:

1. \( xy = 2 \)

2. \( xy + 3 = 0 \)
Assignment:

Pgs 183-190
#1-8, 11, 12, 14, 15, 19 - 21